

Chapter 11 — Rates and Unit Rates — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 45–60 minutes each (see pacing table below)
Materials	Item/price cards, distance and time strips, page cards, two-column rate tables, unit labels, double number lines, the three chapter diagrams
Launch question	“\$18 for 3 notebooks and 240 km in 3 hours — what do these two situations have in common?”
Key representation	The rate table with a labeled one-unit row — keep one visible throughout instruction
Anticipated responses	Correct: divides in the direction that matches the requested units and keeps the unit label attached. Partial: computes the right quotient but drops or mislabels the unit. Incorrect: divides in the wrong direction, or compares totals instead of unit rates.
If the readiness check fails	Reteach division and ratio order from Chapters 3 and 9 before starting the Launch; do not skip the one-unit-group step
Talk prompt	“What should the answer say — per one what?”
Exit ticket	5 items — find a unit rate, compare two deals by unit price, explain why totals mislead, scale from a unit rate, state the routine (see Section 14)
Reteach / extend	Reteach: whole-number rates only, with one-unit physical groups. Extend: decimal and fractional rates, unit conversions, reverse problems.

Purpose and pacing

Chapter 11 connects ratios to quantities with different units. Students must understand that a unit rate is a normalized comparison, not just a decimal answer.

Lesson	Focus	Suggested time
Lesson 1	Readiness, launch, cost and speed rates	45–60 minutes
Lesson 2	Rate tables, double	45–60 minutes

Lesson	Focus	Suggested time
	number lines, and scaling	
Lesson 3	Comparing rates, units, misconceptions, and Worksheet A	45–55 minutes
Lesson 4	Mixed practice, Tripwire, Exit Ticket, and targeted reteaching	50–65 minutes

Materials

Prepare item cards, price cards, distance and time strips, page cards, two-column rate tables, unit labels, and double number lines. Write the requested units before students calculate.

Teacher launch guidance

Begin with \$18 for 3 notebooks and 240 km in 3 hours. Ask students what the two situations have in common. Both require finding the amount for one unit, but the units differ. This establishes that “per” is a general mathematical idea.

When comparing deals, do not allow students to choose based on total price. Require a one-unit row in a table for every comparison.

Teacher prompts

Purpose	Prompt
Identify target units	“What should the answer say per one what?”
Choose division order	“Which quantity belongs in the numerator for those units?”
Make a unit rate	“How can you find the amount for one unit?”
Compare fairly	“Are both rates expressed in the same units?”
Scale from a rate	“What do you multiply by the number of units?”
Check	“Does unit rate times units rebuild the total?”
Diagnose	“What does the student’s answer measure?”

Differentiation

Students needing concrete support

Use small whole-number examples and physically distribute the total among equal groups. Keep the unit card attached to each group. Move from money to speed and pages only after the meaning of “per one” is secure.

Students needing language support

Use the frames:

- “The rate compares _____ with _____.”
- “The requested unit rate is _____ per _____.”
- “I divide _____ by _____ because _____.”
- “The unit rate is _____.”
- “The answer is reasonable because _____ $\times$ _____ = _____.”

Students ready to extend

Use decimal and fractional quantities, unit conversions, rates per minute versus per hour, and inverse problems where the unit rate is given and a total must be found.

Formative assessment

Record whether students identify the requested units, divide in the correct direction, retain units, compare like units, and use inverse multiplication. A correct numerical quotient with incorrect units should be marked developing rather than secure.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: Find the unit rate for 180 miles in 3 hours. Explain the units in \$4.50 for 3 pounds. Compare 3 km in 20 minutes with 5 km in 30 minutes. State when a unit rate is useful.

Strategy Studio guidance: Expected ideas: $3 \text{ km}/20 \text{ min} = 9 \text{ km/h}$; $5 \text{ km}/30 \text{ min} = 10 \text{ km/h}$; the taxi total has a fixed intercept, so it is not proportional; $\$ = \4 each and $\$ = \3.75 each, so the second option is cheaper per item. Keep units attached to every rate.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: $180 \div 3 = 60$ miles per hour. Altered: $3 \text{ km}/20 \text{ min} = 9 \text{ km/h}$ and $5 \text{ km}/30 \text{ min} = 10 \text{ km/h}$. Non-routine: total taxi cost is $C = 4m + 5$, with a fixed intercept. Repair: $\$ = \4 per item, while $\$ = \3.75 per item, so the second package is cheaper per item.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

Ready-to-Learn Check

1. For each one.
2. \$4 per notebook.
3. Divide kilometres by hours.
4. Dollars per hour.
5. \$4 per item.
6. 20 pages.
7. Units identify the meaning of the number.

Worksheet A

1. **\$2/pencil.** Likely wrong answer: 0.5 pencils per dollar (divides pencils by dollars instead of dollars by pencils). Misconception: dividing in the wrong direction — not matching the requested units. Next move: “the answer should say dollars per pencil — which number divides by which?”
2. **\$5/notebook.** Likely wrong answer: 0.2 notebooks per dollar (same reversed-direction error). Misconception: same as #1. Next move: same check.
3. **\$12/h.** Likely wrong answer: 0.083 hours per dollar (same reversed-direction error). Misconception: same as #1. Next move: same check.
4. **50 km/h.** Likely wrong answer: 0.02 hours per km (same reversed-direction error). Misconception: same as #1. Next move: same check.
5. **12 pages/day.** Likely wrong answer: 0.083 days per page (same reversed-direction error). Misconception: same as #1. Next move: same check.
6. **\$21.** Likely wrong answer: \$10 (adds 3 and 7 instead of multiplying the unit rate by the number of pens). Misconception: confusing scaling

with combining. Next move: “does the cost of 7 pens come from adding or multiplying?”

7. **60 km.** Likely wrong answer: 23 km (adds 20 and 3 instead of multiplying). Misconception: same as #6. Next move: “does distance come from adding the speed and time, or multiplying them?”
8. **3 items for \$9, because it is \$3 per item compared with \$4 per item.** Likely wrong answer: picks 2 items for \$8, reasoning that \$8 is a lower total price. Misconception: comparing totals instead of unit rates. Next move: “what does one item cost in each deal?”
9. **Units as stated in answers 1-5.** Common flaw: gives correct numbers but omits the units, or attaches the wrong unit label. Next move: ask “what does this number actually measure?”
10. **The package sizes differ, so compare cost per item.** Common flaw: says “because the numbers are different” without explaining that different package sizes make totals an unfair comparison. Next move: ask what would happen if both packages had the same number of items.

Worksheet B

1. **\$3/marker.** Likely wrong answer: 0.33 markers per dollar (divides in the wrong direction). Misconception: not matching the division order to the requested units. Next move: “the answer should say dollars per marker — which number divides by which?”
2. **\$4/kg.** Likely wrong answer: 0.25 kg per dollar (same reversed-direction error). Misconception:

- same as #1. Next move: same check.
3. **\$13/h.** Likely wrong answer: 0.077 hours per dollar (same reversed-direction error). Misconception: same as #1. Next move: same check.
 4. **70 km/h.** Likely wrong answer: 0.014 hours per km (same reversed-direction error). Misconception: same as #1. Next move: same check.
 5. **15 pages/day.** Likely wrong answer: 0.067 days per page (same reversed-direction error). Misconception: same as #1. Next move: same check.
 6. **30 pages/min.** Likely wrong answer: 0.033 minutes per page (same reversed-direction error). Misconception: same as #1. Next move: same check.
 7. **\$15.** Likely wrong answer: \$8.50 (adds 2.5 and 6 instead of multiplying). Misconception: confusing scaling with combining. Next move: “does the cost of 6 kg come from adding or multiplying?”
 8. **270 km in 3 h is 90 km/h; 400 km in 5 h is 80 km/h, so the first is faster.** Likely wrong answer: picks 400 km as faster, reasoning that 400 is a larger total distance. Misconception: comparing totals instead of unit rates. Next move: “how far does each travel in exactly one hour?”
 9. **72 pages.** Likely wrong answer: 18 pages (adds 6 and 12 instead of multiplying). Misconception: same as #7. Next move: same check.
 10. **300 km.** Likely wrong answer: 79 km (adds 75 and 4 instead of multiplying). Misconception: same as #7. Next move: same check.

11. **A rate compares two quantities with different units; a unit rate describes the amount for one unit.** Common flaw: uses the two terms interchangeably without stating the distinction. Next move: ask for one example of each.
12. **Answers vary but must include different units and a per-one result.** Common flaw: creates a money-based example despite the instruction to avoid money. Next move: check that the units aren’t dollars.

Worksheet C

1. **\$3/bottle.** Likely wrong answer: 0.33 bottles per dollar (divides in the wrong direction). Misconception: not matching the division order to the requested units. Next move: “the answer should say dollars per bottle — which number divides by which?”
2. **Store B at \$2.70/bottle.** Likely wrong answer: picks Store A, reasoning that \$24 is a lower total price than \$27. Misconception: comparing totals instead of unit prices. Next move: “what does one bottle cost at each store?”
3. **9 km/h.** Likely wrong answer: 0.11 hours per km (reversed-direction division). Misconception: not matching the division order to the requested units. Next move: “the answer should say kilometres per hour — which number divides by which?”
4. **45 km.** Likely wrong answer: 14 km (adds 9 and 5 instead of multiplying the unit rate by the number of hours). Misconception: confusing scaling with combining. Next move: “does distance at a constant speed come from adding or multiplying?”

5. **\$13/h.** Likely wrong answer: 0.077 hours per dollar (reversed-direction division). Misconception: same as #3. Next move: same check.
6. **\$260.** Likely wrong answer: 33 (adds 13 and 20 instead of multiplying). Misconception: same as #4. Next move: same check.
7. **Both are 30 pages/min, so they are equally fast.** Likely wrong answer: picks Printer A because 360 pages is a larger raw total than 270 pages. Misconception: comparing totals instead of unit rates. Next move: “how many pages does each printer make in exactly one minute?”
8. **15 parts/hour.** Likely wrong answer: 0.067 hours per part (reversed-direction division). Misconception: same as #3. Next move: same check.
9. **Answers vary but must compare unit rates.** Common flaw: creates a scenario where the larger total genuinely does happen to be better, without realizing this only works when the number of units is also larger by at least as much. Next move: check the unit rates directly, not just the totals.
10. **Units tell what the number measures and determine the correct division order.** Common flaw: says units are “important for clarity” without explaining that they actually determine which quantity goes in the numerator. Next move: ask how the units would show a division error.
11. **60 parts per hour. The number of shifts (2) and the number of days per week (6) were not needed to find the hourly rate.** Likely wrong answer: works the 2 shifts or 6 days into the calculation somehow. Misconception: assuming every number in a word problem must be used. Next move: “what does the question actually ask for? Which numbers answer that?”
12. **Answers vary; must describe a situation where 90 is a total amount, 6 is the number of units, and 15 is the unit rate, with each meaning stated.** Common flaw: gives a context but doesn’t identify all three roles, or confuses which number is the total and which is the unit rate. Next move: ask the student to finish three sentences — “90 is __,” “6 is __,” “15 is __.”

Extend Your Thinking

1. 15 km/h.
2. 1,600 pages/hour.
3. 60 km in 45 minutes is 80 km/h; 70 km in 1 hour is 70 km/h, so the first is faster.
4. \$3.20/kg.
5. A unit rate can be less than 1 when the amount for one unit is less than one unit of the other measure.
6. Answers vary, such as 12 pages/hour and \$12/hour.

Tripwire and Exit Ticket

Tripwire: Deal B at \$4/bottle.

Exit Ticket: \$4/notebook; 60 km/h;

6 items for \$24 at \$4/item versus 4 items for \$20 at \$5/item, so 6 for \$24 is cheaper per item;

totals use different quantities;

identify the target units, divide to one unit, write units, and check by multiplying back.

